# Grade 9/10 Math Circles
## March 27, 2024
## Probability II - Solutions

## In-Lesson Exercises

1. Dice rolls are independent.
   (a) $P(\text{even}) \cdot P(\text{odd}) = 1/2 \cdot 1/2 = 1/4$
   (b) $P(1) \cdot P\left(1^C\right) = P(1) \cdot (1 - P(1)) = 1/6 \cdot (1 - 1/6) = 5/36$

2. Choosing with replacement is independent.
   (a) $P(\text{red}) \cdot P(\text{green}) = 2/10 \cdot 3/10 = 6/100$
   (b) $P(\text{blue}) \cdot P(\text{blue}) = 5/10 \cdot 5/10 = 25/100$

3. Let's write $D =$ you roll doubles and $S =$ the rolls sum to at least 10.

   There are six ways to roll doubles, out of 36 total rolls, so $P(D) = 6/36 = 1/6$.

   There are two ways to roll doubles that sum to at least 10 (5/5 and 6/6), so $P(D \cap S) = 2/36$.

$$P(S|D) = \frac{P(D \cap S)}{P(D)} = \frac{2/36}{1/6} = 1/3$$

4. Since $B \subseteq A$, we know $A \cap B = B$.

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(B)}{P(B)} = 1$$

5. We already found the probability of getting a pair of red socks in the lesson, so now we just need to find the probability of getting a pair of black socks and a pair of white socks. The probability of getting any pair will be the sum of the probabilities of getting a pair of each colour.

   Let $B1 =$ the first sock is black, $B2 =$ the second sock is black, $W1 =$ the first sock is white, and $W2 =$ the second sock is white.

$$P(B1 \cap B2) = P(B1) \cdot P(B2|B1) = \frac{6}{20} \cdot \frac{5}{19} = \frac{30}{380}$$

$$P(W1 \cap W2) = P(W1) \cdot P(W2|W1) = \frac{4}{20} \cdot \frac{3}{19} = \frac{12}{380}$$

   So, we see that

$$P(\text{pair}) = \frac{30}{380} + \frac{90}{380} + \frac{12}{380} = \frac{132}{380} \approx 35\%$$

6. Let $B$ be the event that someone plays board games and let $V$ be the event that someone plays video games. We know $P(V|B) = 5/10$, $P\left(V|B^C\right) = 2/10$, and $P(B) = 7/10$. The complement rule says $P\left(B^C\right) = 3/10$. So,

$$P(V) = P(V|B) \cdot P(B) + P\left(V|B^C\right) \cdot P\left(B^C\right) = \frac{5}{10} \cdot \frac{7}{10} + \frac{2}{10} \cdot \frac{3}{10} = \frac{41}{100}$$

7. Let $C$ mean someone likes cookies and $B$ mean someone likes brownies. We want to find $P(B|C)$.

   We know that $P(C) = \frac{8}{10}$, $P(B) = \frac{4}{10}$, and $P(C|B) = \frac{9}{10}$. Using Bayes' Theorem,

$$P(B|C) = \frac{P(C|B) \cdot P(B)}{P(C)} = \frac{\frac{9}{10} \cdot \frac{4}{10}}{\frac{8}{10}} = \frac{36}{80} = 0.45$$